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Sets

TOPIC 1 Sets, Types of Sets, Disjoint Sets, Complementary Sets, Subsets, Power Set, Cardinal Number of Sets, Operations on Sets

1. Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is _____.
[Sep. 06, 2020 (I)]

2. Let $S = \{1, 2, 3, \ldots, 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is :
[Jan. 12, 2019 (I)]

(a) $2^{100} - 1$ (b) $2^{50}\left(2^{50} - 1\right)$
(c) $2^{50} - 1$ (d) $2^{50} + 1$

3. Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S : **[2018]**
(a) contains exactly one element.
(b) contains exactly two elements.
(c) contains exactly four elements.
(d) is an empty set.

4. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S: **[2016]**
(a) contains exactly two elements.
(b) contains more than two elements.
(c) is an empty set.
(d) contains exactly one element.

5. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then:
[Online April 10, 2016]

(a) $P \subset Q$ and $Q - P \neq \phi$
(b) $Q \not\subset P$
(c) $P = Q$
(d) $P \not\subset Q$

6. A relation on the set $A = \{x : |x| < 3, x \in Z\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power set of R is: **[Online April 12, 2014]**
(a) 32 (b) 16 (c) 8 (d) 64

7. Let $X = \{1,2,3,4,5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is : **[2012]**
(a) 5^2 (b) 3^5 (c) 2^5 (d) 5^3

8. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[2009]**
(a) $A = C$ (b) $B = C$
(c) $A \cap B = \phi$ (d) $A = B$

TOPIC 2 Venn Diagrams, Algebraic Operations on Sets, De Morgan's Law, Number of Elements in Different Sets

9. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be : **[Sep. 05, 2020 (I)]**
(a) 63 (b) 36 (c) 54 (d) 38

10. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be : **[Sep. 04, 2020 (I)]**
(a) 29 (b) 37 (c) 65 (d) 55

11. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to **[Sep. 04, 2020 (II)]**
(a) 15 (b) 50 (c) 45 (d) 30

12. Let $X = \{n \in N: 1 \le n \le 50\}$. If
$A = \{n \in X: n \text{ is a multiple of } 2\}$ and
$B = \{n \in X: n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is _____. **[Jan. 7, 2020 (II)]**

13. Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set A × B, is : **[Jan. 12, 2019 (II)]**
(a) 2^{15} (b) 2^{18} (c) 2^{12} (d) 2^{10}

14. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is: **[Jan. 10, 2019 (II)]**
(a) 102 (b) 42 (c) 1 (d) 38

15. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true ? **[April 12, 2019 (II)]**
(a) $B \cap C \neq \phi$
(b) If $(A - B) \subseteq C$, then $A \subseteq C$
(c) $(C \cup A) \cap (C \cup B) = C$
(d) If $(A - C) \subseteq B$, then $A \subseteq B$

16. Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is: **[April. 09, 2019 (II)]**
(a) 13.9 (b) 12.8 (c) 13 (d) 13.5

17. In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements : **[Online April 10, 2015]**
(A) 5% families own both a car and a phone
(B) 35% families own either a car or a phone
(C) 40,000 families live in the town
Then,
(a) Only (A) and (C) are correct.
(b) Only (B) and (C) are correct.
(c) All (A), (B) and (C) are correct.
(d) Only (A) and (B) are correct.

Hints & Solutions

1. **(28)** $2^m = 112 + 2^n \Rightarrow 2^m - 2^n = 112$

$\Rightarrow 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$

$\therefore m = 7,\ n = 4 \Rightarrow mn = 28$

2. **(b)** $\because$ Product of two even number is always even and product of two odd numbers is always odd.

$\therefore$ Number of required subsets

= Total number of subsets – Total number of subsets having only odd numbers

$= 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

3. **(b)** **Case-I:** $x \in [0,9]$

$2(3 - \sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 4, 2$

$\Rightarrow x = 16, 4$

Since $x \in [0,9]$

$\therefore \quad x = 4$

Case-II: $x \in [9,\infty]$

$2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$

Since $x \in [9,\infty]$

$\therefore x = 16$

Hence, $x = 4$ & 16

4. **(a)** $f(x) + 2f\left(\frac{1}{x}\right) = 3x \quad ....(1)$

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x} \quad ....(2)$

Adding (1) and (2)

$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = x + \frac{1}{x} \quad ...(3)$

Substracting (1) from (2)

$\Rightarrow f(x) - f\left(\frac{1}{x}\right) = \frac{3}{x} - 3x \quad ...(4)$

On adding (3) and (4)

$\Rightarrow f(x) = \frac{2}{x} - x$

$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x \Rightarrow x = \frac{2}{x}$

$x^2 = 2 \quad$ or $\quad x = \sqrt{2}, -\sqrt{2}$

5. **(c)** $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \cos\theta + \sqrt{2}\cos\theta$

$= (\sqrt{2} + 1)\cos\theta = \left(\frac{2-1}{\sqrt{2}-1}\right)\cos\theta$

$\Rightarrow (\sqrt{2} - 1)\sin\theta = \cos\theta$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\therefore \quad P = Q$

6. **(b)** $A = \{x : |x| < 3,\ x \in Z\}$

$A = \{-2, -1, 0, 1, 2\}$

$R = \{(x, y) : y = |x|,\ x \neq -1\}$

$R = \{(-2, 2), (0, 0), (1, 1), (2, 2)\}$

R has four elements

Number of elements in the power set of R

$= 2^4 = 16$

7. **(b)** Let $X = \{1, 2, 3, 4, 5\}$

$n(x) = 5$

Each element of x has 3 options. Either in set Y or set Z or none. ($\because Y \cap Z = \phi$)

So, number of ordered pairs $= 3^5$

8. **(b)** $\because \quad B = (B \cap A) \cup B$

$= (A \cap C) \cup B$

$= (A \cup B) \cap (C \cup B)$

$= (A \cup C) \cap (B \cup C)$

$= (A \cap B) \cup C$

$= (A \cap C) \cup C$

$= C$

9. **(b)** Given, $n(C) = 73,\ n(T) = 65,\ n(C \cap T) = x$

$\therefore 65 \geq n(C \cap T) \geq 65 + 73 - 100$

$\Rightarrow 65 \geq x \geq 38 \Rightarrow x \neq 36.$

10. **(d)** Let $n(U) = 100$, then $n(A) = 63, n(B) = 76$

$n(A \cap B) = x$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B) \leq 100$

$= 63 + 76 - x \leq 100$

$\Rightarrow x \geq 139 - 100 \Rightarrow x \geq 39$

$\because n(A \cap B) \le n(A)$

$\Rightarrow x \le 63$

$\therefore 39 \le x \le 63$

11. **(d)** $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$

$\because \quad n(X_i) = 10,\ n(Y_i) = 5$

So, $\bigcup_{i=1}^{50} X_i = 500, \bigcup_{i=1}^{n} Y_i = 5n$

$\Rightarrow \frac{500}{20} = \frac{5n}{6} \Rightarrow n = 30$

12. **(29)** From the given conditions,

$n(A) = 25, n(B) = 7$ and $n(A \cap B) = 3$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 25 + 7 - 3 = 29$

13. **(a)** Let $x \in A$, then

$\because \quad 2^{(x+2)(x^2-5x+6)} = 1 \Rightarrow (x+2)(x-2)(x-3) = 0$

$x = -2, 2, 3$

$A = \{-2, 2, 3\}$

Then, $n(A) = 3$

Let $x \in B$, then

$-3 < 2x - 1 < 9$

$-1 < x < 5$ and $x \in Z$

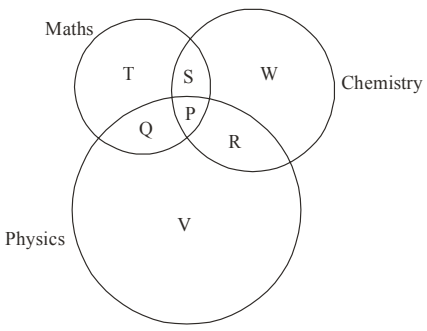
$\therefore \quad B = \{0, 1, 2, 3, 4\}$

$n(B) = 5$

$n(A \times B) = 3 \times 5 = 15$

Hence, Number of subsets of $A \times B = 2^{15}$

14. **(d)**



$P = \{30, 60, 90, 120\}$

$\Rightarrow \quad n(P) = 4$

$Q = \{6n : n \in N, 1 \le n \le 23\} - P$

$\Rightarrow \quad n(Q) = 19$

$R = \{15n : n \in N, 1 \le n \le 9\} - P$

$\Rightarrow \quad n(R) = 5$

$S = \{10n : n \in N, 1 \le n \le 14\} - P$

$\Rightarrow \quad n(S) = 10$

$n(T) = 70 - n(P) - n(Q) - n(S) = 70 - 33 = 37$

$n(V) = 46 - n(P) - n(Q) - n(R) = 46 - 28 = 18$

$n(W) = 28 - n(P) - n(R) - n(S) = 28 - 19 = 9$

$\Rightarrow$ Number of required students

$= 140 - (4 + 19 + 5 + 10 + 37 + 18 + 9)$

$= 140 - 102 = 38$

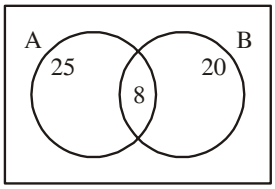
15. **(d)** (1), (2) and (4) are always correct.

In (3) option,

If $A = C$ then $A - C = \phi$

Clearly, $\phi \subseteq B$ but $A \subseteq B$ is not always true.

16. **(a)**



% of people who reads A only = 25 – 8 = 17%

% of people who read B only = 20 – 8 = 12%

% of people from A only who read advertisement

$= 17 \times 0.3 = 5.1\%$

% of people from B only who read advertisement

$= 12 \times 0.4 = 4.8\%$

% of people from A & B both who read advertisement

$= 8 \times 0.5 = 4\%$

$\therefore$ total % of people who read advertisement

$= 5.1 + 4.8 + 4 = 13.9\%$

17. **(c)** n(P) = 25%

n(C) = 15%

$n(P' \cup C') = 65\%$

$\Rightarrow n(P \cup C)' = 65\%$

$n(P \cup C) = 35\%$

$n(P \cap C) = n(P) + n(C) - n(P \cup C)$

$25 + 15 - 35 = 5\%$

$x \times 5\% = 2000$

$x = 40{,}000$